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LETTER TO THE EDITOR

On the possibility of stable quark and pion-condensed stars

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Abstract. The conditions for the appearance of stable quark stars and pion-condensed stars are analysed. On the basis of recently suggested equations of state such configurations can exist either as a third stable family or as a separated part of the stable neutron star branch.

During the last decade several suggestions have been made concerning the high-density state of nuclear matter. At present it seems that a phase transition to quark matter is not avoidable in the ultrahigh region (cf Chapline and Nauenberg 1977, Baluni 1978, Keister and Kisslinger 1976). At densities somewhat beyond the nuclear density a phase transition to a pion-condensed state of the nuclear matter is expected (Migdal 1978). The consequences for the static neutron star parameters (like mass and radius) have been studied extensively. Hartle *et al* (1975) claimed the division of the stable neutron star branch in two parts. Bowers *et al* (1977) showed the cut-off of the stable branch by a sufficiently strong phase transition. Several authors (cf Bowers *et al* 1977, Baym and Chin 1976, Anand *et al* 1979) advocated the non-existence of stable quark stars as a separated part beyond the neutron star peak. From most calculations it turned out that the critical density for the phase transition from nuclear matter to quark matter is not reached inside stable neutron stars (Källmann 1980, Alvarez 1981, Morley and Kislinger 1979). Henceforth one had to ask for the conditions of developing a stable quark star branch beyond the neutron star peak.

In the case of a phase transition (to quark or pion-condensed matter) within the stable neutron star region one must ask for changing the stability properties of these configurations. That means one tries to investigate the question of whether the stars with a phase transition are a part of the ordinary neutron star branch or that of a separated stable branch. In answering these questions one must bear in mind the uncertainties of our knowledge in both the subnuclear and the supernuclear density range. As a result of this there are no commonly accepted values of the critical density as well as of the strength of the phase transitions. Therefore, it is reasonable to use convenient *ad hoc* equations of state in which a phase transition is approximated by a step or a jump. It is the aim of this Letter to present the general features of the mass–radius relation if the strength and the on-set of the phase transition, as well as the continuing equation of state, is varied. The purpose in doing so is to find out under what conditions a third stable branch (as part of the neutron star peak or not) arises. This third stable branch offers the possibility of special supernova-like collapse events in which an unstable configuration at the end of the ordinary stable neutron star branch settles down in a new stable quark or pion-condensed state (Migdal *et al* 1979, Haensel

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and Proszynski 1980). Besides the modified neutron star cooling (Maxwell 1978) such special collapse events would give strongest hints for the real existence for extraordinary states of ultradense matter.

To discuss the influence of a step or a jump in the equation of state on the static neutron star properties we choose a model equation of state in the form

$$P = K\alpha\rho(\rho/\rho_0)^\alpha \tag{1}$$

with $K = 0.2$, $\rho_0 = 10^{15}$ g cm^{-3}, $\alpha = 0.5$ ($c = 1$ is used). With this choice one approximates currently accepted equations of state quite well.

Further, such a form is useful for calculating the collapse dynamics (cf Van Riper 1980, Lichtenstadt *et al* 1980, Shapiro and Teukolsky 1980).

For continuing the equation of state beyond the onset of the phase transition at the critical pressure P_0 and the corresponding density ρ_1 let us choose two extreme cases. Firstly

$$P = P_0 + A(\rho - \rho_1) \qquad \text{for } \rho_1 < \rho < \lambda\rho_1 \tag{2a}$$

$$P = P_0 + A\rho_1(\lambda - 1) + K\alpha(\rho/\lambda)(\rho/\lambda\rho_0)^\alpha \qquad \text{for } \rho > \lambda\rho_1. \tag{2b}$$

The step (2*a*) reflects the strong softening during the phase transition. A is chosen to be fixed at 10^{-3}. The following continuation is relatively soft, as suggested by the σ-model calculations for the pion-condensate, e.g. by Weise and Brown (1975). The opposite case is a nearly incompressible continuation for which one uses for convenience

$$\rho = \lambda\rho_1 \qquad \text{for } P > P_0. \tag{3}$$

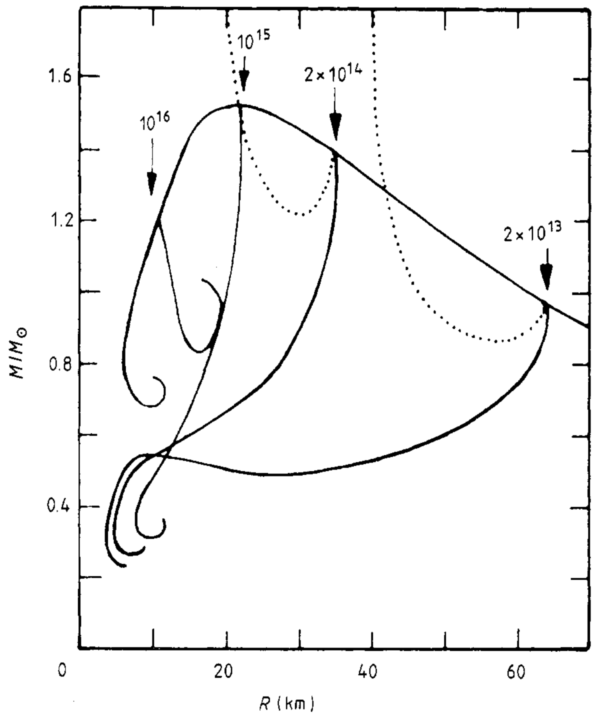

Figure 1. The mass–radius plot for $\lambda = 2.0$ and several critical densities (in g cm^{-3}) using the equation of state (2). The dotted lines use the equation of state (3).

This is suggested by quark–matter calculations, e.g. by Bowers *et al* (1977). Of course, the stiff continuation only holds in a narrow range, then the equation of state approches the causality limit.

The types (2) and (3) certainly cover the actual behaviour. The influence of several reasonable choices of the free parameters λ and P_0 on the mass–radius plot is displayed in figures 1–3 for relativistic cold spherical stars determined by the TOV equations.

Figure 1 shows the mass–radius plot for $\lambda = 2.0$ and several critical densities at which the phase transition happens. As is well known (Lighthill 1950, Kämpfer 1981) for this value of λ, a cusp appears in the plot (not visible for $\rho_1 = 10^{16}$ g cm^{-3} in the scale chosen). The cusp is a counterclockwise passed extremum where an unstable mode arises (Bardeen *et al* 1966, Hartle *et al* 1975). Because of the soft continuation of the equation of state, configurations with the phase transition at $\rho_1 = 2\times 10^{14}$ g cm^{-3} remain unstable. Only configurations with the phase transition at sufficiently low densities retain their stability at the occurring minimum. As shown by Bowers *et al* (1977) a strong phase transition on the unstable branch cannot remove the fundamental instability. Soft phase transitions with $\lambda = 1.3$ are demonstrated in figure 2. An instability arises, however, of a type other than the cusp instability found by Ramsey (1950) and Lighthill (1950). Here, this instability appears for finite-density cores. It results because of the enhanced compressibility. The stabilising effects of relativity found previously (Kämpfer 1981) cover only a narrow range followed by the occurrence of an instability even for small λ values. Even for configurations sufficiently near to the top of the neutron star peak, no stable high-density branch develops (seen for $\rho_1 = 2\times 10^{14}$ g cm^{-3} in figure 2).

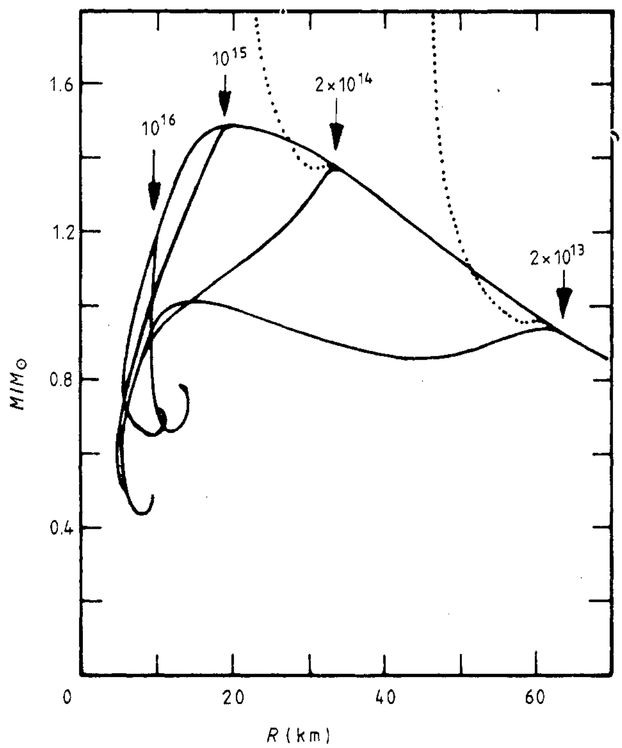

Figure 2. The same as in figure 1 with $\lambda = 1.3$.

The situation is in contrast to Newtonian theory where in every case the stable high-density branch develops. In general relativity one observes that the high-density continuation must be stiff enough in order to allow stable quark or pion-condensed stars as a separated part of the neutron star branch. This is demonstrated by the incompressible continuation (3) of the equation of state by dotted lines in the figures 1 and 2.

Particular attention is devoted to the phase transition in configurations beyond the stable neutron star branch. Gerlach (1968) claimed that a sudden stiffening of the equation of state gives rise to a separated third stable family. However, no hint of such a stiffening at high densities is detected. But the very stiff or nearly incompressible continuation after a phase transition is, in general, accepted. It produces the third family or the proper quark or pion-condensed island (figure 3, configurations on the left side of the minimum). The condition for this is that only a narrow-range phase transition occurs. For wide-range phase transitions the cusp phenomenon will appear, giving rise to an unstable mode; this appears for $\lambda > 1.8$.

The condition for the real existence of the proper quark or pion-condensed stars in nature is quite subtle: the continuation of the equation of state beyond the weak phase transition must be stiff enough over a sufficiently wide pressure region to generate the minimum in figure 3 signalling the on-set of a stable branch.

Let us summarise the results obtained: (i) an instability on the stable neutron star branch already occurs for jump parameters $\lambda < \lambda_{\text{crit}} = 1.5\ (1 + P_0/\rho_1)$, (ii) the unstable

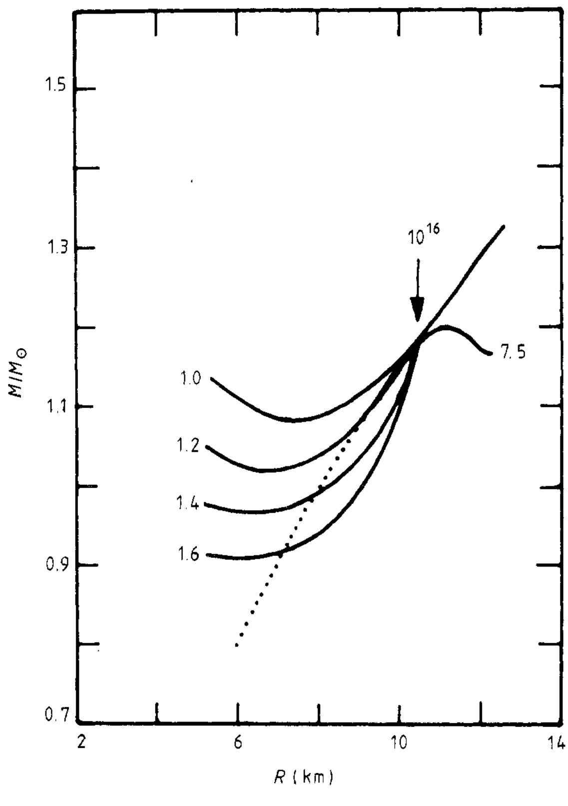

Figure 3. The mass–radius plot using the equation of state (3) with several values of λ. The minima are reached at the central pressures $3P_0$ ($\lambda = 1.0$), $4P_0$ ($\lambda = 1.2$), $6P_0$ ($\lambda = 1.4$) and $8P_0$ ($\lambda = 1.6$). The dotted line shows the ordinary continuation of the equation of state.

mode arising only disappears for the continuing equation of state being sufficiently stiff, or when the on-set of the phase transition is sufficiently far from the maximum of the neutron star peak, (iii) a proper third stable branch rises beyond the neutron star peak if the continuing equation of state is nearly incompressible over a sufficiently large pressure range and $\lambda < \lambda_{\mathrm{crit}}$ holds. Whether the points (i), (ii) or (iii) are fulfilled (therefore allowing stable quark or pion-condensed stars) or not must be decided by further work.

In every case during the collapse of a neutron star being unstable due to a phase transition, the density discontinuity runs through the star as a shock wave heating the matter behind it.

Calculations using the Ramsey model show that the shock velocity is of the right order of magnitude in order to blow off the outermost shell via the shock heating mechanism of Lichtenstadt *et al* (1980). If there is a third stable branch one expects the development of the bounce-off shock wave which heats the matter once more and promotes the blowing off of the mantle. For checking this picture one has to perform collapse calculations with discontinuous equations of state. Such calculations are in progress (Migdal A B 1980, private communication).

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